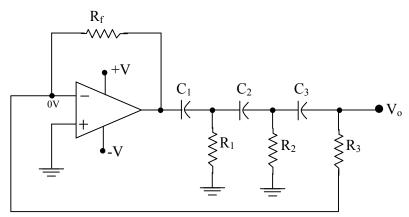
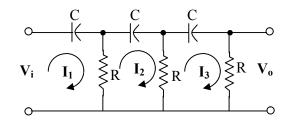
The Phase-Shift Oscillator:

The following figure shows the circuit diagram of the phase-shift oscillator. Oscillation occurs at the frequency where the total phase shift through the three RC feedback circuits is 180° . The inversion of the op-amp itself provides the another 180° phase shift to meet the requirement for oscillation of a 360° (or 0°) phase shift around the feedback loop.



The feedback circuit in the phase-shift oscillator is shown in the following figure. In the derivation we assume,

 $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C_3 = C$



Using mesh analysis we have,

$$(R + 1/j\omega C)I_1 - RI_2 = V_i \qquad \dots \dots (1) - RI_1 + (2R + 1/j\omega C)I_2 - RI_3 = 0 \qquad \dots \dots (2) - RI_2 + (2R + 1/j\omega C)I_3 \qquad \dots \dots (3)$$

In order to get V_0 , we must solve for I_3 using determinants:

$$I_{3} = \frac{\begin{vmatrix} (R + 1/j\omega C) & -R & V_{i} \\ -R & (2R + 1/j\omega C) & 0 \\ 0 & -R & 0 \end{vmatrix}}{\begin{vmatrix} (R + 1/j\omega C) & -R & 0 \\ -R & (2R + 1/j\omega C) & -R \\ 0 & -R & (2R + 1/j\omega C) \end{vmatrix}}$$
$$= \frac{R^{2}V_{i}}{(R + 1/j\omega C)[(2R + 1/j\omega C)^{2} - R^{2}] - R^{2}(2R + 1/j\omega C)}$$

Now,
$$\frac{V_0}{V_i} = \frac{RI_3}{V_i} = \frac{R^3}{(R+1/j\omega C)[(2R+1/j\omega C)^2 - R^2] - R^2(2R+1/j\omega C)}$$
$$= \frac{R^3}{(R+1/j\omega C)(2R+1/j\omega C)^2 - R^2(R+1/j\omega C) - R^2(2R+1/j\omega C)}$$
$$= \frac{1}{(1+1/j\omega RC)(2+1/j\omega RC)^2 - (1+1/j\omega RC) - (2+1/j\omega RC)}$$
$$= \frac{1}{(1+1/j\omega RC)(4+4/j\omega RC - 1/\omega^2 R^2 C^2) - (3+2/j\omega RC)}$$
$$= \frac{1}{(4+4/j\omega RC - 1/\omega^2 R^2 C^2 + 4/j\omega RC - 4/\omega^2 R^2 C^2 - 1/j\omega^3 R^3 C^3) - 3 - 2/j\omega RC)}$$
$$= \frac{1}{(1-5/\omega^2 R^2 C^2 + 6/j\omega RC - 1/j\omega^3 R^3 C^3)}$$
$$= \frac{1}{(1-5/\omega^2 R^2 C^2) - j(6/\omega RC - 1/\omega^3 R^3 C^3)} \dots \dots (4)$$

For oscillation in the phase-shift amplifier, the phase shift through the RC circuit must be equal to 180°. For this condition to exist, the j term must be 0 at the frequency of oscillation ω_0 . $\Rightarrow 6/\omega_0 RC - 1/\omega_0^3 R^3 C^3 = 0$

$$\Rightarrow \frac{6\omega_0^2 R^2 C^2 - 1}{\omega_0^3 R^3 C^3} = 0$$
$$\Rightarrow \frac{6\omega_0^2 R^2 C^2 - 1}{\omega_0^3 R^3 C^3} = 0$$

$$\Rightarrow \omega_0^2 = \frac{1}{6R^2C^2}$$
$$\Rightarrow \omega_0 = \frac{1}{RC\sqrt{6}}$$
$$\Rightarrow f_0 = \frac{1}{2\pi RC\sqrt{6}}$$

Now, from the equation (4) we have,

$$\frac{V_0}{V_1} = \frac{1}{(1-5x6)} = -\frac{1}{29}$$

The negative sign results from the 180° inversion by the circuit. Thus, the value of voltage gain by the RC circuit is,

$$\frac{V_0}{V_i} = \frac{1}{29}$$

To meet the greater-than-unity loop gain requirement, the closed-loop voltage gain of the op-amp must be greater than 29.

So, $R_f \ge 29 R_3$

Exercise:

- (a) Determine the value of R_f necessary for the circuit in Figure 17–14 to operate as an oscillator.
- (b) Determine the frequency of oscillation.

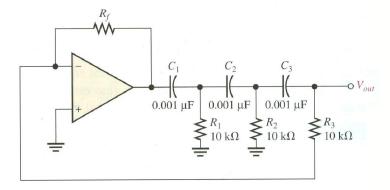


FIGURE 17–14

Solution

(a)
$$A_{cl} = 29$$
, and $B = \frac{1}{29} = \frac{R_3}{R_f}$. Therefore,
 $\frac{R_f}{R_3} = 29$
 $R_f = 29R_3 = 29(10 \text{ k}\Omega) = 290 \text{ k}\Omega$
(b) $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C_3 = C$. Therefore,

$$f_r = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(10 \text{ k}\Omega)(0.001 \text{ }\mu\text{F})} \cong 6.5 \text{ kHz}$$

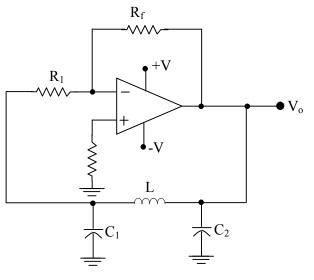
Related Exercise

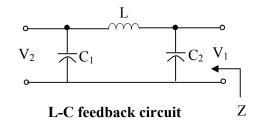
- (a) If R_1 , R_2 , and R_3 in Figure 17–14 are changed to 8.2 k Ω , what value must R_f be for oscillation?
- (b) What is the value of f_r ?

Open file FG17-14.CA4 on your circuit disk. Measure the frequency of oscillation and compare to the calculated value.

The Colpitts Oscillator:

The following figure shows the circuit diagram of the Colpitts oscillator. Oscillation occurs at the frequency where the L-C feedback circuits is at resonance.





Colpitts oscillator

Assuming $R_1 \gg X_{C1}$ we have the impedance of the L-C circuit,

$$Z = \frac{(-jX_{C2})(jX_{L} - jX_{C1})}{(-jX_{C2} + jX_{L} - jX_{C1})}$$
$$Z = \frac{X_{C2}(X_{L} - X_{C1})}{j(X_{L} - X_{C2} - X_{C1})}$$

At parallel resonance the impedance will be maximum and we can write,

$$\begin{split} X_{L} - X_{C2} - X_{C1} &= 0 \\ \Rightarrow X_{L} - X_{C1} &= X_{C2} \quad \dots \dots (1) \\ \Rightarrow \omega L - 1/\omega C_{1} &= 1/\omega C_{2} \\ \Rightarrow \omega L &= 1/\omega C_{1} + 1/\omega C_{2} = \frac{1}{\omega} \frac{C_{1} + C_{2}}{C_{1}C_{2}} \\ \Rightarrow \omega^{2} &= \frac{1}{L} \frac{1}{C_{1}C_{2}/(C_{1} + C_{2})} \\ \Rightarrow \omega^{2} &= \frac{1}{\sqrt{LC_{1}C_{2}/(C_{1} + C_{2})}} \\ = \frac{1}{\sqrt{LC_{T}}} \qquad \text{where } C_{T} = \frac{C_{1}C_{2}}{C_{1} + C_{2}} \\ \Rightarrow f &= \frac{1}{2\pi\sqrt{LC_{T}}} \end{split}$$

Again, the voltage gain of the LC circuit,

$$\frac{V_2}{V_1} = \frac{-jX_{C1}}{jX_L - jX_{C1}} = \frac{-X_{C1}}{X_L - X_{C1}}$$

Here negative sign is for 180° phase shift by the circuit. So magnitude of the voltage gain is,

$$\beta = \frac{X_{C1}}{X_L - X_{C1}}$$

$$= \frac{X_{C1}}{X_{C2}} \qquad (\text{from equation (1)})$$

$$= \frac{C_2}{C_1}$$

For oscillation to sustain, the loop gain must be greater than unity. Therefore, the voltage gain of the amplifier should be,

 $|\mathbf{A}_{v}| > \frac{1}{\beta}$ $\Rightarrow \frac{\mathbf{Rf}}{\mathbf{R1}} > \frac{\mathbf{C}_{1}}{\mathbf{C}_{2}}$

Example: Design of OP-AMP Colpitts Oscillator

Design the Colpitts oscillator to produce a 40 kHz output frequency. Use a 100 mH inductor and an OP-AMP with a ± 10 V supply.

SOLUTION We know,

$$C_{\rm T} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (40 \text{ kHz})^2 \times 100 \text{ mH}} = 153.8 \text{ pF}$$

For $C_1 \approx 10C_2$, $C_1 \approx 10C_T = 10 \times 153.8 \text{ pF} = 1538 \text{ pF}$ (use 1500 pF standard value)

$$C_{2} = \frac{1}{(1/C_{T}) - (1/C_{1})} = \frac{1}{(1/153.8 \text{ pF}) - (1/1500 \text{ pF})}$$

= 177 pF (use 180 pF standard value)
$$X_{C2} = \frac{1}{2\pi f C_{2}} = \frac{1}{2\pi \times 40 \text{ kHz} \times 180 \text{ pF}} = 22 \text{ k}\Omega$$
$$X_{C2} >> Z_{0} \text{ of the amplifier}$$
$$X_{C1} = \frac{1}{2\pi f C_{1}} = \frac{1}{2\pi \times 40 \text{ kHz} \times 1500 \text{ pF}} = 2.65 \text{ k}\Omega$$

Since $R_1 >> X_{C1}$, we select

$$R_1 = 10X_{C1} = 10 \times 2.65 \text{ k}\Omega = 26.5 \text{ k}\Omega \text{ (use } 27 \text{ k}\Omega \text{ standard value)}$$

Now $A_{\text{CL(min)}} = \frac{C_1}{C_2} = \frac{1500 \text{ pF}}{180 \text{ pF}} = 8.33$ $R_2 = A_{\text{CL(min)}}R_1 = 8.33 \times 27 \text{ k}\Omega = 225 \text{ k}\Omega \text{ (use } 270 \text{ k}\Omega \text{ standard value)}$ $R_3 = R_1 \|R_2 = 27 \text{ k}\Omega\| 270 \text{ k}\Omega = 24.5 \text{ k}\Omega \text{ (use } 27 \text{ k}\Omega \text{ standard value)}$

The OP-AMP full-power bandwidth (f_p) must be a minimum of 40 kHz when $V_0 \approx \pm 9$ V and $A_{CL} = 8.33$.

Since $f_2 = A_{CL} \times f_p$, therefore,

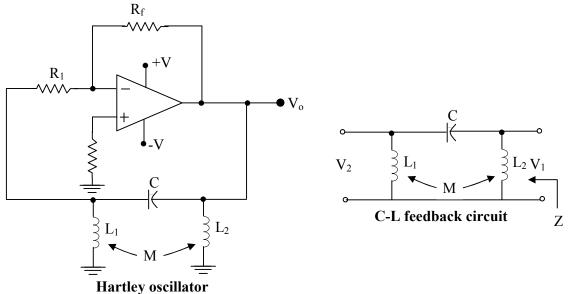
$$f_2 = 8.33 \times 40 \text{ kHz} = 333 \text{ kHz}$$

and

Slew rate,
$$SR = 2\pi f_p V_p = 2\pi \times 40 \text{ kHz} \times 9 \text{ V} = 2.262 \text{ V}/\mu s$$

The Hartley Oscillator:

The following figure shows the circuit diagram of the Hartley oscillator. Oscillation occurs at the frequency where the C-L feedback circuits is at resonance.



Assuming $R_1 \gg X_{L1}$ we have the impedance of the C-L circuit,

$$Z = \frac{(jX_{L2} + jX_{M})(-jX_{C} + jX_{L1} + jX_{M})}{(jX_{L2} + jX_{M} - jX_{C} + jX_{L1} + jX_{M})}$$
$$Z = \frac{(X_{L2} + X_{M})(-X_{C} + X_{L1} + X_{M})}{j(X_{L2} - X_{C} + X_{L1} + 2X_{M})}$$

At parallel resonance the impedance will be maximum and we can write,

$$X_{L2} - X_{C} + X_{L1} + 2X_{M} = 0$$

$$\Rightarrow X_{L1} + X_{M} - X_{C} = -(X_{L2} + X_{M}) \quad \dots \dots (1)$$

$$\Rightarrow \omega L_{1} + 2\omega M - 1/\omega C = -\omega L_{2}$$

$$\Rightarrow \omega L_{1} + \omega L_{2} + 2\omega M = 1/\omega C$$

$$\Rightarrow \omega^{2} = \frac{1}{C} \frac{1}{(L_{1} + L_{2} + 2M)}$$

$$\Rightarrow \omega = \frac{1}{\sqrt{C(L_{1} + L_{2} + 2M)}}$$

$$= \frac{1}{\sqrt{CL_{T}}} \qquad \text{where } L_{T} = L_{1} + L_{2} + 2M$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{CL_{T}}}$$

Again, the voltage gain of the C-L circuit,

$$\frac{V_2}{V_1} = \frac{jX_{L1} + jX_M}{jX_{L1} + jX_M - jX_C} = \frac{X_{L1} + X_M}{X_{L1} + X_M - X_C}$$
$$\Rightarrow \beta = \frac{X_{L1} + X_M}{X_{L1} + X_M - X_C}$$
$$= -\frac{X_{L1} + X_M}{X_{L2} + X_M}$$
(from equation (1))

Here negative sign is for 180° phase shift by the circuit. So magnitude of the voltage gain is,

$$\beta = \frac{X_{L1} + X_M}{X_{L2} + X_M}$$
$$\Rightarrow \beta = \frac{L_1 + M}{L2 + M}$$

For oscillation to sustain, the loop gain must be greater than unity. Therefore, the voltage gain of the amplifier should be,

$$|A_{v}| > \frac{1}{\beta}$$
$$\Rightarrow \frac{Rf}{R1} > \frac{L_{2} + M}{L_{1} + M}$$

If the inductors are wound on separate core, then mutual inductance M = 0 and we can write,

 $\frac{\mathrm{Rf}}{\mathrm{R1}} > \frac{\mathrm{L_2}}{\mathrm{L_1}}$

Example: Design of OP-AMP Hartley Oscillator

Design the Hartley oscillator to produce a 100 kHz output frequency with an amplitude of ± 8 V. For simplicity, assume that there is no mutual inductance between L_1 and L_2 .

SOLUTION
$$V_{CC} = \pm (V_0 + 1 \text{ V}) = \pm (8 \text{ V} + 1 \text{ V}) = \pm 9 \text{ V}$$

 $X_{L2} >> Z_0 \text{ of the amplifier}$

Select $X_{L2} \approx 1 \,\mathrm{k}\Omega$

$$X_{L2} = \frac{X_{L2}}{2\pi f} = \frac{1 \,\mathrm{k}\Omega}{2\pi \times 100 \,\mathrm{kHz}} = 1.59 \,\mathrm{mH} \,\mathrm{(use} \,1.5 \,\mathrm{mH} \,\mathrm{standard} \,\mathrm{value})$$

Select $L_1 \approx \frac{L_2}{10} = \frac{1.5 \text{ mH}}{10} = 150 \ \mu\text{H} \text{ (standard value)}$ $L_T = L_1 + L_2 = 1.5 \text{ mH} + 150 \ \mu\text{H} = 1.65 \text{ mH}$

Now,

$$C = \frac{1}{4\pi^2 f^2 L_{\rm T}} = \frac{1}{4\pi^2 \times (100 \text{ kHz})^2 \times 1.65 \text{ mH}}$$

= 1535 pF (use 1500 pF with additional parallel capacitance, if necessary)
 $C >> \text{ stray capacitance}$

$$X_{\rm L1} = 2\pi f L_1 = 2\pi \times 100 \,\rm kHz \times 150 \,\mu\rm H = 94.2 \,\Omega$$

 $R_1 >> X_{L1}$ Select $R_1 = 1 \text{ k}\Omega$ (standard value)

Therefore,

$$A_{\rm CL(min)} = \frac{L_2}{L_1} = \frac{1.5 \,\mathrm{mH}}{150 \,\mu\mathrm{H}} = 10$$

$$R_2 = A_{\text{CL(min)}}R_1 = 10 \times 1 \,\text{k}\Omega = 10 \,\text{k}\Omega \,(\text{standard value})$$

 $R_3 = R_1 || R_2 = 1 \text{ k}\Omega || 10 \text{ k}\Omega = 909 \Omega \text{ (use } 1 \text{ k}\Omega \text{ standard value)}$

The OP-AMP full-power bandwidth (f_p) must be a minimum of 100 kHz when $V_0 \approx \pm 8$ V and $A_{CL} = 10$.

Since $f_2 = A_{CL} \times f_p$, therefore,

 $f_2 = 10 \times 100 \text{ kHz} = 1 \text{ MHz}$

Slew rate, $SR = 2\pi f_p V_p = 2\pi \times 100 \text{ kHz} \times 8 \text{ V} = 5 \text{ V}/\mu s$

and

The following figures show the circuit diagram of the Wein Bridge oscillator. Oscillation occurs at the particular frequency when ac balance is obtained in the Wein Bride. At the balanced condition of the bridge we can write,

$$\begin{split} \frac{Z_2 V_0}{Z_1 + Z_2} &= \frac{Z_4 V_0}{Z_3 + Z_4} \\ \Rightarrow \frac{Z_2}{Z_1 + Z_2} &= \frac{Z_4}{Z_3 + Z_4} \\ \Rightarrow \frac{(R_2)(1/j\omega C_2)/(R_2 + 1/j\omega C_2)}{(R_1 + 1/j\omega C_1) + (R_2)(1/j\omega C_2)/(R_2 + 1/j\omega C_2)} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{(R_2)(1/j\omega C_2)}{(R_1 + 1/j\omega C_1)(R_2 + 1/j\omega C_2) + (R_2)(1/j\omega C_2)} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_1} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_2} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_2} = \frac{R_4}{R_3 + R_4} \\ \Rightarrow \frac{j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2) + j\omega R_2 C_2} = \frac{R_4}{R_3 + R_4} \\ \end{cases}$$

Since, the right hand side of the above equation is a real term, the left hand side must also be a real term. So, we can write,

$$1 - \omega^2 R_1 C_1 R_2 C_2 = 0$$

$$\Rightarrow \omega = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}} \dots \dots (2)$$

From equation (1) we have,

$$\Rightarrow \frac{R_{2}C_{1}}{(R_{1}C_{1} + R_{2}C_{2} + R_{2}C_{1})} = \frac{R_{4}}{R_{3} + R_{4}}$$
$$\Rightarrow \frac{R_{2}C_{1}}{R_{1}C_{1} + R_{2}C_{2}} = \frac{R_{4}}{R_{3}}$$

$$\Rightarrow \frac{R_3}{R_4} = \frac{R_1C_1 + R_2C_2}{R_2C_1} = \frac{R_1}{R_2} + \frac{C_2}{C_1} \quad \dots \dots (3)$$

The op-amp along with the two resistors R_3 and R_4 constitutes a non-inverting amplifier who's voltage gain is,

$$A_{\rm V} = 1 + \frac{R_3}{R_4}$$

Using the of R_3/R_4 obtained in equation (3) we have,

$$A_{v} = 1 + \frac{R_{1}}{R_{2}} + \frac{C_{2}}{C_{1}}$$

This corresponds that the attenuation of the feedback network is,

$$1/\left(1 + \frac{R_1}{R_2} + \frac{C_2}{C_1}\right)$$

Therefore, A_V must be equal to or greater than $\left(1 + \frac{R_1}{R_2} + \frac{C_2}{C_1}\right)$ to sustain oscillation. Mathematically,

$$A_{v} \ge \left(1 + \frac{R_{1}}{R_{2}} + \frac{C_{2}}{C_{1}}\right) \dots \dots (4)$$

$$\Rightarrow 1 + \frac{R_{3}}{R4} \ge \left(1 + \frac{R_{1}}{R_{2}} + \frac{C_{2}}{C_{1}}\right)$$

$$\Rightarrow \frac{R_{3}}{R_{4}} \ge \frac{R_{1}}{R_{2}} + \frac{C_{2}}{C_{1}}$$

For R₁ = R₂ = R and C₁ = C₂ = C we have,

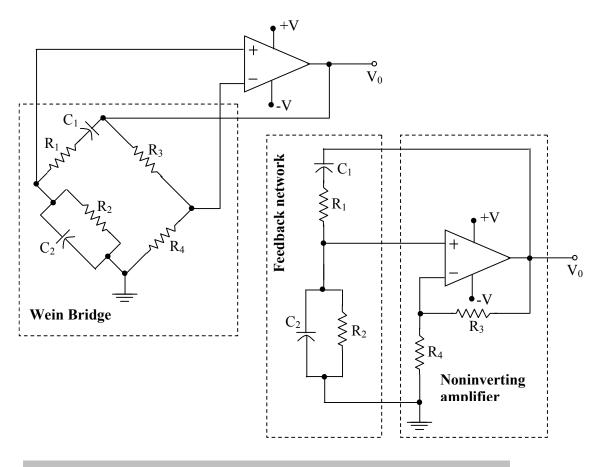
$$\frac{R_3}{R_4} \ge 2$$

$$\Rightarrow R_3 \ge 2R_4$$

Also from equation (4) we have,
 $A_V \ge 3$

From equation (2) we have,

$$\omega = \frac{1}{RC}$$
$$\Rightarrow f = \frac{1}{2\pi RC}$$



Example: Design of Wein Bridge Oscillator

Design the Wein bridge oscillator to produce a 100 kHz output frequency with an amplitude of ± 9 V. Design the amplifier to have a closed-loop gain of 3.

SOLUTION $V_{\rm CC} = \pm (V_0 + 1 \text{ V}) = \pm (9 \text{ V} + 1 \text{ V}) = \pm 10 \text{ V}$

For
$$A_{CL} = 3$$
,
Also,
Select,
 $R_1 = R_2 = R$ and $C_1 = C_2 = C$
 $R_3 = 2R_4$
 $C_1 = 1000 \text{ pF} (\text{standard value})$
 $C_2 = C_1 = 1000 \text{ pF}$

Therefore, $R = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 100 \text{ kHz} \times 1000 \text{ pF}}$ = 1.59 k Ω (use 1.5 k Ω standard value) Select, $R_4 \approx R_2 = 1.5 \text{ k}\Omega$ (standard value)

 $R_3 = R_4 = 2 \times 1.5 \text{ k}\Omega = 3 \text{ k}\Omega \text{ (use 3.3 k}\Omega \text{ standard value)}$

The OP-AMP full-power bandwidth (f_p) must be a minimum of 100 kHz when $V_0 \approx \pm 9$ V and $A_{CL} = 3$.

Since $f_2 = A_{CL} \times f_p$, therefore,

 $f_2 = 3 \times 100 \text{ kHz} = 300 \text{ kHz}$

and

Slew rate, $SR = 2\pi f_p V_p = 2\pi \times 100 \text{ kHz} \times 9 \text{ V} \approx 5.7 \text{ V}/\mu s$

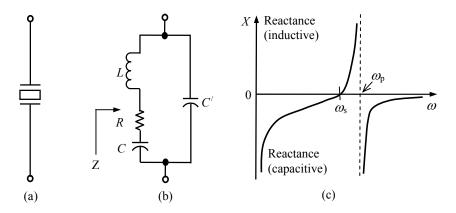


Figure 9.25 A piezoelectric crystal: (a) Symbol, (b) electrical model, and (c) the reactance function (if R = 0).

9.12 CRYSTAL OSCILLATORS

If a piezoelectric crystal, usually quartz, has electrodes plated on opposite faces and if a potential is applied between these electrodes, forces will be exerted on the bound charges within the crystal. If this device is properly mounted, deformations take place within the crystal, and an electromechanical system is formed which will vibrate when properly excited. The resonant frequency and the Q depend upon the crystal dimensions, how the surfaces are oriented with respect to its axes, and how the device is mounted. Frequencies ranging from a few kilohertz to a few megahertz, and Q's in the range from several thousand to several hundred thousand, are commercially available. These extraordinarily high values of Q and the fact that the characteristics of quartz are extremely stable with respect to time and temperature account for the exceptional frequency stability of oscillators incorporating crystals. Crystal oscillators are used whenever great stability is required, for example, in communication transmitters and receivers.

The electrical equivalent circuit of a crystal is indicated in Figure 9.25. The inductor L, capacitor C, and resistor R are the analogs of the mass, the compliance (the reciprocal of the spring constant), and the viscous-damping factor of the mechanical system. The typical values for a 90-kHz crystal are L = 137 H, C = 0.0235 pF, and R = 15 kΩ, corresponding to Q = 5,500. The dimensions of such a crystal are 30 by 4 by 1.5 mm. Since C' represents the electrostatic capacitance between electrodes with the crystal as a dielectric, its magnitude (~3.5 pF) is very much larger than C.

If we neglect the resistance R, the impedance of the crystal (Z in Figure 9.25(b)) is given by,

$$Z = Z_1 || Z_2$$

$$\Rightarrow \qquad = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\Rightarrow \qquad = \frac{(-j1/\omega C')(j\omega L - j1/\omega C)}{j\omega L - j1/\omega C - j1/\omega C'}$$

$$\Rightarrow \qquad = -\frac{j}{\omega C'} \frac{L(\omega - 1/\omega LC)}{L(\omega - 1/\omega LC - 1/\omega LC')}$$

$$\Rightarrow \qquad = -\frac{j}{\omega C'} \frac{\omega^2 - 1/LC}{\omega^2 - \frac{1}{L}(1/C + 1/C')}$$

$$\Rightarrow \qquad = -\frac{j}{\omega C'} \frac{\omega^2 - \omega_{\rm s}^2}{\omega^2 - \omega_{\rm p}^2} \tag{9.64}$$

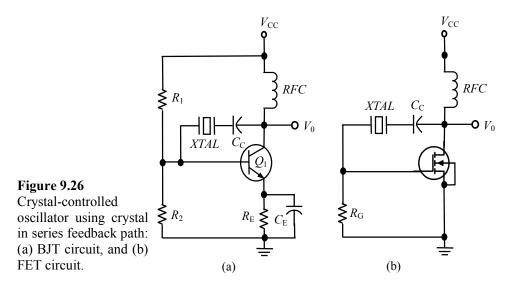
Where $\omega_{\rm s} = \frac{1}{\sqrt{LC}}$ = series resonant frequency and $\omega_{\rm p} = \sqrt{\frac{1}{L} \left(\frac{1}{C} + \frac{1}{C'}\right)}$ = parallel resonant frequency. Equation (9.64) can be written, in terms of reactance, as

$$Z = jX = -\frac{j}{\omega C'} \frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2}$$
(9.65)

Therefore, reactance of the crystal is

$$X = -\frac{1}{\omega C'} \frac{\omega^2 - \omega_{\rm s}^2}{\omega^2 - \omega_{\rm p}^2}$$
(9.66)

The plot of Equation (9.66) is shown in Figure 9.25(c). Since C' >> C, $\omega_p \approx \omega_s$. For the crystal whose parameters are specified above, the parallel frequency is only three-tenths of 1 percent higher than the series frequency. For $\omega_s < \omega < \omega_p$, the reactance is inductive, and outside this range it is capacitive, as indicated in Figure 9.25(c). In order to use the crystal properly it must be connected in a circuit so that its low impedance in the series resonant operating mode or high impedance in the parallel resonant operating mode is selected.

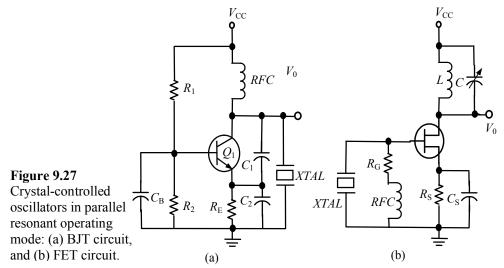


Series Resonant Circuits

To excite a crystal for operation in the series resonant mode it may be connected as a series element in a feedback path. At the series resonant frequency of the crystal its impedance is smallest and the amount of (positive) feedback is largest. A typical transistor circuit is shown in Figure 9.26. Resistors R_1 , R_2 , and R_E provide a voltage divider stabilized *dc* bias circuit.

Capacitor C_E provides *ac* bypass of the emitter resistor and the *RFC* coil provides for *dc* bias while decoupling any *ac* signal on the power lines from affecting the output signal. The voltage feedback from collector to base is a maximum when the crystal (*XTAL*) impedance is minimum (in series resonant mode). The coupling capacitor C_C has negligible impedance at the circuit operating frequency but blocks and dc between collector and base.

The resulting circuit frequency of oscillation is set by the series resonant frequency of the crystal. Changes in supply voltage, transistor device parameters, and so on, have no effect on the circuit operating frequency which is held stabilized by the crystal. The circuit frequency stability is set by the crystal frequency stability, which is good.



Parallel Resonant Circuits

Since the parallel resonant impedance of a crystal is a maximum value, it is connected in parallel/shunt. At the parallel resonant operating frequency a crystal appears as an inductive reactance of largest value. Figure 9.27(a) shows a crystal connected as the inductor connected in a modified Colpitts circuit. The basic dc bias circuit should be evident. Maximum voltage is developed across the crystal at its parallel resonant frequency. The voltage is coupled to the emitter by a capacitor voltage divider — capacitors C_1 and C_2 .

A *Miller crystal-controlled oscillator* circuit is shown in Figure 9.27(b). A tuned *LC* circuit in the drain section is adjusted near the crystal parallel resonant frequency. The maximum gate-source signal occurs at the crystal parallel resonant frequency controlling the circuit operating frequency.

OP-AMP Crystal Oscillator

An OP-AMP can be used in a crystal oscillator as shown in Figure 9.28. The crystal is connected in the series resonant path and operates at the crystal series resonant frequency. The present circuit has a high gain so that an output square-wave signal results as shown in the figure. A pair of Zener diodes is shown at the output to provide output amplitude at exactly the Zener voltage (V_Z) .

