## The Phase-Shift Oscillator:

The following figure shows the circuit diagram of the phase-shift oscillator. Oscillation occurs at the frequency where the total phase shift through the three RC feedback circuits is $180^{\circ}$. The inversion of the op-amp itself provides the another $180^{\circ}$ phase shift to meet the requirement for oscillation of a $360^{\circ}$ (or $0^{\circ}$ ) phase shift around the feedback loop.


The feedback circuit in the phase-shift oscillator is shown in the following figure. In the derivation we assume,
$\mathrm{R}_{1}=\mathrm{R}_{2}=\mathrm{R}_{3}=\mathrm{R}$ and $\mathrm{C}_{1}=\mathrm{C}_{2}=\mathrm{C}_{3}=\mathrm{C}$


Using mesh analysis we have,

$$
\begin{align*}
& (\mathrm{R}+1 / \mathrm{j} \omega \mathrm{C}) \mathrm{I}_{1}-\mathrm{RI}_{2}=\mathrm{V}_{\mathrm{i}}  \tag{1}\\
& -\mathrm{RI}_{1}+(2 \mathrm{R}+1 / \mathrm{j} \omega \mathrm{C}) \mathrm{I}_{2}-\mathrm{RI}_{3}=0  \tag{2}\\
& -\mathrm{RI}_{2}+(2 \mathrm{R}+1 / \mathrm{j} \omega \mathrm{C}) \mathrm{I}_{3} \tag{3}
\end{align*}
$$

In order to get $\mathrm{V}_{0}$, we must solve for $\mathrm{I}_{3}$ using determinants:

$$
\begin{aligned}
I_{3} & =\frac{\left|\begin{array}{ccc}
(R+1 / j \omega C) & -R & V_{i} \\
-R & (2 R+1 / j \omega C) & 0 \\
0 & -R & 0
\end{array}\right|}{\left\lvert\, \begin{array}{ccc}
(R+1 / j \omega C) & -R & 0 \\
-R & (2 R+1 / j \omega C) & -R \\
0 & -R & (2 R+1 / j \omega C)
\end{array}\right.} \\
& =\frac{R^{2} V_{i}}{(R+1 / j \omega C)\left[(2 R+1 / j \omega C)^{2}-R^{2}\right]-R^{2}(2 R+1 / j \omega C)}
\end{aligned}
$$

$$
\begin{align*}
& \text { Now, } \frac{V_{0}}{V_{i}}=\frac{R I_{3}}{V_{i}}=\frac{R^{3}}{(R+1 / j \omega C)\left[(2 R+1 / j \omega C)^{2}-R^{2}\right]-R^{2}(2 R+1 / j \omega C)} \\
& =\frac{R^{3}}{(R+1 / j \omega C)(2 R+1 / j \omega C)^{2}-R^{2}(R+1 / j \omega C)-R^{2}(2 R+1 / j \omega C)} \\
& =\frac{1}{(1+1 / j \omega R C)(2+1 / j \omega R C)^{2}-(1+1 / j \omega R C)-(2+1 / j \omega R C)} \\
& =\frac{1}{(1+1 / j \omega R C)\left(4+4 / j \omega R C-1 / \omega^{2} R^{2} C^{2}\right)-(3+2 / j \omega R C)} \\
& =\frac{1}{\left.\left(4+4 / j \omega R C-1 / \omega^{2} R^{2} C^{2}+4 / j \omega R C-4 / \omega^{2} R^{2} C^{2}-1 / j \omega^{3} R^{3} C^{3}\right)-3-2 / j \omega R C\right)} \\
& =\frac{1}{\left(1-5 / \omega^{2} R^{2} C^{2}+6 / j \omega R C-1 / j \omega^{3} R^{3} C^{3}\right)} \\
& =\frac{1}{\left(1-5 / \omega^{2} R^{2} C^{2}\right)-j\left(6 / \omega R C-1 / \omega^{3} R^{3} C^{3}\right)} \ldots \ldots(4) \tag{4}
\end{align*}
$$

For oscillation in the phase-shift amplifier, the phase shift through the RC circuit must be equal to $180^{\circ}$. For this condition to exist, the j term must be 0 at the frequency of oscillation $\omega_{0}$.
$\Rightarrow 6 / \omega_{0} \mathrm{RC}-1 / \omega_{0}^{3} \mathrm{R}^{3} \mathrm{C}^{3}=0$
$\Rightarrow \frac{6 \omega_{0}^{2} R^{2} C^{2}-1}{\omega_{0}^{3} R^{3} C^{3}}=0$
$\Rightarrow 6 \omega_{0}^{2} \mathrm{R}^{2} \mathrm{C}^{2}-1=0$
$\Rightarrow \omega_{0}^{2}=\frac{1}{6 \mathrm{R}^{2} \mathrm{C}^{2}}$
$\Rightarrow \omega_{0}=\frac{1}{\mathrm{RC} \sqrt{6}}$
$\Rightarrow \mathrm{f}_{0}=\frac{1}{2 \pi \mathrm{RC} \sqrt{6}}$
Now, from the equation (4) we have,
$\frac{\mathrm{V}_{0}}{\mathrm{~V}_{\mathrm{i}}}=\frac{1}{(1-5 \times 6)}=-\frac{1}{29}$
The negative sign results from the $180^{\circ}$ inversion by the circuit. Thus, the value of voltage gain by the RC circuit is,
$\frac{\mathrm{V}_{0}}{\mathrm{~V}_{\mathrm{i}}}=\frac{1}{29}$
To meet the greater-than-unity loop gain requirement, the closed-loop voltage gain of the op-amp must be greater than 29 .

So, $\mathrm{R}_{\mathrm{f}} \geq 29 \mathrm{R}_{3}$

## Exercise:

(a) Determine the value of $R_{f}$ necessary for the circuit in Figure 17-14 to operate as an oscillator.
(b) Determine the frequency of oscillation.


FIGURE 17-14

## Solution

(a) $A_{c l}=29$, and $B=\frac{1}{29}=\frac{R_{3}}{R_{f}}$. Therefore,

$$
\begin{aligned}
\frac{R_{f}}{R_{3}} & =29 \\
R_{f} & =29 R_{3}=29(10 \mathrm{k} \Omega)=\mathbf{2 9 0} \mathbf{~} \boldsymbol{\Omega}
\end{aligned}
$$

(b) $R_{1}=R_{2}=R_{3}=R$ and $C_{1}=C_{2}=C_{3}=C$. Therefore,

$$
f_{r}=\frac{1}{2 \pi \sqrt{6} R C}=\frac{1}{2 \pi \sqrt{6}(10 \mathrm{k} \Omega)(0.001 \mu \mathrm{~F})} \cong \mathbf{6 . 5} \mathbf{~ k H z}
$$

## Related Exercise

(a) If $R_{1}, R_{2}$, and $R_{3}$ in Figure 17-14 are changed to $8.2 \mathrm{k} \Omega$, what value must $R_{f}$ be for oscillation?
(b) What is the value of $f_{r}$ ?

Open file FG17-14.CA4 on your circuit disk. Measure the frequency of oscillation and compare to the calculated value.

## The Colpitts Oscillator:

The following figure shows the circuit diagram of the Colpitts oscillator. Oscillation occurs at the frequency where the L-C feedback circuits is at resonance.


L-C feedback circuit

## Colpitts oscillator

Assuming $\mathrm{R}_{1} \gg \mathrm{X}_{\mathrm{C} 1}$ we have the impedance of the L-C circuit,
$\mathrm{Z}=\frac{\left(-\mathrm{j} \mathrm{X}_{\mathrm{C} 2}\right)\left(\mathrm{j} \mathrm{X}_{\mathrm{L}}-\mathrm{j} \mathrm{X}_{\mathrm{C} 1}\right)}{\left(-\mathrm{j} \mathrm{X}_{\mathrm{C} 2}+\mathrm{j} \mathrm{X}_{\mathrm{L}}-\mathrm{j} \mathrm{X}_{\mathrm{C} 1}\right)}$
$\mathrm{Z}=\frac{\mathrm{X}_{\mathrm{C} 2}\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C} 1}\right)}{\mathrm{j}\left(\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C} 2}-\mathrm{X}_{\mathrm{C} 1}\right)}$
At parallel resonance the impedance will be maximum and we can write,
$\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C} 2}-\mathrm{X}_{\mathrm{C} 1}=0$
$\Rightarrow \mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{C} 1}=\mathrm{X}_{\mathrm{C} 2}$
$\Rightarrow \omega \mathrm{L}-1 / \omega \mathrm{C}_{1}=1 / \omega \mathrm{C}_{2}$
$\Rightarrow \omega \mathrm{L}=1 / \omega \mathrm{C}_{1}+1 / \omega \mathrm{C}_{2}=\frac{1}{\omega} \frac{\mathrm{C}_{1}+\mathrm{C}_{2}}{\mathrm{C}_{1} \mathrm{C}_{2}}$
$\Rightarrow \omega^{2}=\frac{1}{\mathrm{~L}} \frac{1}{\mathrm{C}_{1} \mathrm{C}_{2} /\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right)}$
$\Rightarrow \omega=\frac{1}{\sqrt{\mathrm{LC}_{1} \mathrm{C}_{2} /\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right)}}$
$=\frac{1}{\sqrt{\mathrm{LC}_{\mathrm{T}}}} \quad$ where $\mathrm{C}_{\mathrm{T}}=\frac{\mathrm{C}_{1} \mathrm{C}_{2}}{\mathrm{C}_{1}+\mathrm{C}_{2}}$
$\Rightarrow \mathrm{f}=\frac{1}{2 \pi \sqrt{\mathrm{LC}_{\mathrm{T}}}}$
Again, the voltage gain of the LC circuit,
$\frac{V_{2}}{V_{1}}=\frac{-j X_{C 1}}{j X_{L}-j X_{C 1}}=\frac{-X_{C 1}}{X_{L}-X_{C 1}}$
Here negative sign is for $180^{\circ}$ phase shift by the circuit. So magnitude of the voltage gain is,
$\beta=\frac{\mathrm{X}_{\mathrm{C} 1}}{\mathrm{X}_{\mathrm{L}}-\mathrm{X}_{\mathrm{Cl}}}$
$=\frac{\mathrm{X}_{\mathrm{C} 1}}{\mathrm{X}_{\mathrm{C} 2}} \quad \quad$ (from equation (1))
$=\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}$
For oscillation to sustain, the loop gain must be greater than unity. Therefore, the voltage gain of the amplifier should be,
$\left|\mathrm{A}_{\mathrm{v}}\right|>\frac{1}{\beta}$
$\Rightarrow \frac{\mathrm{Rf}}{\mathrm{R} 1}>\frac{\mathrm{C}_{1}}{\mathrm{C}_{2}}$

## Example: Design of OP-AMP Colpitts Oscillator

Design the Colpitts oscillator to produce a 40 kHz output frequency. Use a 100 mH inductor and an OP-AMP with a $\pm 10 \mathrm{~V}$ supply.

SOLUTION We know,

$$
C_{\mathrm{T}}=\frac{1}{4 \pi^{2} f^{2} L}=\frac{1}{4 \pi^{2} \times(40 \mathrm{kHz})^{2} \times 100 \mathrm{mH}}=153.8 \mathrm{pF}
$$

For $C_{1} \approx C_{2}, C_{1} \approx 10 C_{\mathrm{T}}=10 \times 153.8 \mathrm{pF}=1538 \mathrm{pF}$ (use 1500 pF standard value)

$$
\begin{aligned}
& C_{2}=\frac{1}{\left(1 / C_{\mathrm{T}}\right)-\left(1 / C_{1}\right)}=\frac{1}{(1 / 153.8 \mathrm{pF})-(1 / 1500 \mathrm{pF})} \\
&=177 \mathrm{pF}(\text { use } 180 \mathrm{pF} \text { standard value }) \\
& X_{\mathrm{C} 2}=\frac{1}{2 \pi f C_{2}}=\frac{1}{2 \pi \times 40 \mathrm{kHz} \times 180 \mathrm{pF}}=22 \mathrm{k} \Omega \\
& X_{\mathrm{C} 2} \gg Z_{0} \text { of the amplifier } \\
& X_{\mathrm{C} 1}=\frac{1}{2 \pi f C_{1}}=\frac{1}{2 \pi \times 40 \mathrm{kHz} \times 1500 \mathrm{pF}}=2.65 \mathrm{k} \Omega
\end{aligned}
$$

Since $R_{1} \gg X_{C 1}$, we select

$$
R_{1}=10 X_{C 1}=10 \times 2.65 \mathrm{k} \Omega=26.5 \mathrm{k} \Omega(\text { use } 27 \mathrm{k} \Omega \text { standard value })
$$

Now $A_{\mathrm{CL}(\text { min })}=\frac{C_{1}}{C_{2}}=\frac{1500 \mathrm{pF}}{180 \mathrm{pF}}=8.33$

$$
\begin{aligned}
& R_{2}=A_{\mathrm{CL}(\text { min })} R_{1}=8.33 \times 27 \mathrm{k} \Omega=225 \mathrm{k} \Omega \text { (use } 270 \mathrm{k} \Omega \text { standard value) } \\
& \quad R_{3}=R_{1}\left\|R_{2}=27 \mathrm{k} \Omega\right\| 270 \mathrm{k} \Omega=24.5 \mathrm{k} \Omega \text { (use } 27 \mathrm{k} \Omega \text { standard value) }
\end{aligned}
$$

The OP-AMP full-power bandwidth $\left(f_{\mathrm{p}}\right)$ must be a minimum of 40 kHz when $V_{0} \approx \pm 9 \mathrm{~V}$ and $A_{\mathrm{CL}}=8.33$.

Since $f_{2}=A_{\mathrm{CL}} \times f_{\mathrm{p}}$, therefore,

$$
f_{2}=8.33 \times 40 \mathrm{kHz}=333 \mathrm{kHz}
$$

and
Slew rate, $S R=2 \pi f_{\mathrm{p}} V_{\mathrm{p}}=2 \pi \times 40 \mathrm{kHz} \times 9 \mathrm{~V}=2.262 \mathrm{~V} / \mu \mathrm{s}$

## The Hartley Oscillator:

The following figure shows the circuit diagram of the Hartley oscillator. Oscillation occurs at the frequency where the C-L feedback circuits is at resonance.



C-L feedback circuit

Assuming $R_{1} \gg X_{L 1}$ we have the impedance of the C-L circuit,
$Z=\frac{\left(j X_{L 2}+j X_{M}\right)\left(-j X_{C}+j X_{L 1}+j X_{M}\right)}{\left(j X_{L 2}+j X_{M}-j X_{C}+j X_{L 1}+j X_{M}\right)}$
$Z=\frac{\left(X_{L 2}+X_{M}\right)\left(-X_{C}+X_{L 1}+X_{M}\right)}{j\left(X_{L 2}-X_{C}+X_{L 1}+2 X_{M}\right)}$
At parallel resonance the impedance will be maximum and we can write,
$\mathrm{X}_{\mathrm{L} 2}-\mathrm{X}_{\mathrm{C}}+\mathrm{X}_{\mathrm{L} 1}+2 \mathrm{X}_{\mathrm{M}}=0$
$\Rightarrow \mathrm{X}_{\mathrm{L} 1}+\mathrm{X}_{\mathrm{M}}-\mathrm{X}_{\mathrm{C}}=-\left(\mathrm{X}_{\mathrm{L} 2}+\mathrm{X}_{\mathrm{M}}\right)$
$\Rightarrow \omega \mathrm{L}_{1}+2 \omega \mathrm{M}-1 / \omega \mathrm{C}=-\omega \mathrm{L}_{2}$
$\Rightarrow \omega \mathrm{L}_{1}+\omega \mathrm{L}_{2}+2 \omega \mathrm{M}=1 / \omega \mathrm{C}$
$\Rightarrow \omega^{2}=\frac{1}{\mathrm{C}} \frac{1}{\left(\mathrm{~L}_{1}+\mathrm{L}_{2}+2 \mathrm{M}\right)}$
$\Rightarrow \omega=\frac{1}{\sqrt{\mathrm{C}\left(\mathrm{L}_{1}+\mathrm{L}_{2}+2 \mathrm{M}\right)}}$
$=\frac{1}{\sqrt{\mathrm{CL}_{\mathrm{T}}}} \quad$ where $\mathrm{L}_{\mathrm{T}}=\mathrm{L}_{1}+\mathrm{L}_{2}+2 \mathrm{M}$
$\Rightarrow \mathrm{f}=\frac{1}{2 \pi \sqrt{\mathrm{CL}_{\mathrm{T}}}}$
Again, the voltage gain of the C-L circuit,
$\frac{V_{2}}{V_{1}}=\frac{j X_{L 1}+j X_{M}}{j X_{L 1}+j X_{M}-j X_{C}}=\frac{X_{L 1}+X_{M}}{X_{L 1}+X_{M}-X_{C}}$
$\Rightarrow \beta=\frac{\mathrm{X}_{\mathrm{L} 1}+\mathrm{X}_{\mathrm{M}}}{\mathrm{X}_{\mathrm{L} 1}+\mathrm{X}_{\mathrm{M}}-\mathrm{X}_{\mathrm{C}}}$
$=-\frac{X_{L 1}+X_{M}}{X_{L 2}+X_{M}}$

Here negative sign is for $180^{\circ}$ phase shift by the circuit. So magnitude of the voltage gain is,
$\beta=\frac{X_{L 1}+X_{M}}{X_{L 2}+X_{M}}$
$\Rightarrow \beta=\frac{\mathrm{L}_{1}+\mathrm{M}}{\mathrm{L} 2+\mathrm{M}}$
For oscillation to sustain, the loop gain must be greater than unity. Therefore, the voltage gain of the amplifier should be,
$\left|A_{v}\right|>\frac{1}{\beta}$
$\Rightarrow \frac{\mathrm{Rf}}{\mathrm{R} 1}>\frac{\mathrm{L}_{2}+\mathrm{M}}{\mathrm{L}_{1}+\mathrm{M}}$
If the inductors are wound on separate core, then mutual inductance $\mathrm{M}=0$ and we can write, $\frac{\mathrm{Rf}}{\mathrm{R} 1}>\frac{\mathrm{L}_{2}}{\mathrm{~L}_{1}}$

## Example: Design of OP-AMP Hartley Oscillator

Design the Hartley oscillator to produce a 100 kHz output frequency with an amplitude of $\pm 8 \mathrm{~V}$. For simplicity, assume that there is no mutual inductance between $L_{1}$ and $L_{2}$.

## SOLUTION $\quad V_{\mathrm{CC}}= \pm\left(V_{0}+1 \mathrm{~V}\right)= \pm(8 \mathrm{~V}+1 \mathrm{~V})= \pm 9 \mathrm{~V}$

$$
X_{\mathrm{L} 2} \gg Z_{0} \text { of the amplifier }
$$

Select $X_{\mathrm{L} 2} \approx 1 \mathrm{k} \Omega$

$$
X_{\mathrm{L} 2}=\frac{X_{\mathrm{L} 2}}{2 \pi f}=\frac{1 \mathrm{k} \Omega}{2 \pi \times 100 \mathrm{kHz}}=1.59 \mathrm{mH} \text { (use } 1.5 \mathrm{mH} \text { standard value) }
$$

Select $L_{1} \approx \frac{L_{2}}{10}=\frac{1.5 \mathrm{mH}}{10}=150 \mu \mathrm{H}($ standard value $)$

$$
L_{\mathrm{T}}=L_{1}+L_{2}=1.5 \mathrm{mH}+150 \mu \mathrm{H}=1.65 \mathrm{mH}
$$

Now,

$$
\begin{aligned}
C & =\frac{1}{4 \pi^{2} f^{2} L_{\mathrm{T}}}=\frac{1}{4 \pi^{2} \times(100 \mathrm{kHz})^{2} \times 1.65 \mathrm{mH}} \\
& =1535 \mathrm{pF} \text { (use } 1500 \mathrm{pF} \text { with additional parallel capacitance, if necessary) } \\
C & \gg \text { stray capacitance }
\end{aligned}
$$

$$
X_{\mathrm{L} 1}=2 \pi f L_{1}=2 \pi \times 100 \mathrm{kHz} \times 150 \mu \mathrm{H}=94.2 \Omega
$$

$$
R_{1} \gg X_{\mathrm{L} 1}
$$

Select $R_{1}=1 \mathrm{k} \Omega$ (standard value)
Therefore,

$$
A_{\mathrm{CL}(\min )}=\frac{L_{2}}{L_{1}}=\frac{1.5 \mathrm{mH}}{150 \mu \mathrm{H}}=10
$$

$$
R_{2}=A_{\mathrm{CL}(\min )} R_{1}=10 \times 1 \mathrm{k} \Omega=10 \mathrm{k} \Omega \text { (standard value) }
$$

$$
R_{3}=R_{1}\left\|R_{2}=1 \mathrm{k} \Omega\right\| 10 \mathrm{k} \Omega=909 \Omega(\text { use } 1 \mathrm{k} \Omega \text { standard value })
$$

The OP-AMP full-power bandwidth $\left(f_{\mathrm{p}}\right)$ must be a minimum of 100 kHz when $V_{0} \approx \pm 8 \mathrm{~V}$ and $A_{\mathrm{CL}}=10$.

Since $f_{2}=A_{\mathrm{CL}} \times f_{\mathrm{p}}$, therefore,

$$
f_{2}=10 \times 100 \mathrm{kHz}=1 \mathrm{MHz}
$$

and

$$
\text { Slew rate, } S R=2 \pi f_{\mathrm{p}} V_{\mathrm{p}}=2 \pi \times 100 \mathrm{kHz} \times 8 \mathrm{~V}=5 \mathrm{~V} / \mu \mathrm{S}
$$

## Wein Bridge Oscillator:

The following figures show the circuit diagram of the Wein Bridge oscillator. Oscillation occurs at the particular frequency when ac balance is obtained in the Wein Bride. At the balanced condition of the bridge we can write,

$$
\begin{align*}
& \frac{Z_{2} V_{0}}{Z_{1}+Z_{2}}=\frac{Z_{4} V_{0}}{Z_{3}+Z_{4}} \\
& \Rightarrow \frac{Z_{2}}{Z_{1}+Z_{2}}=\frac{Z_{4}}{Z_{3}+Z_{4}} \\
& \Rightarrow \frac{\left(R_{2}\right)\left(1 / j \omega C_{2}\right) /\left(R_{2}+1 / j \omega C_{2}\right)}{\left(R_{1}+1 / j \omega C_{1}\right)+\left(R_{2}\right)\left(1 / j \omega C_{2}\right) /\left(R_{2}+1 / j \omega C_{2}\right)}=\frac{R_{4}}{R_{3}+R_{4}} \\
& \Rightarrow \frac{\left(R_{2}\right)\left(1 / j \omega C_{2}\right)}{\left(R_{1}+1 / j \omega C_{1}\right)\left(R_{2}+1 / j \omega C_{2}\right)+\left(R_{2}\right)\left(1 / j \omega C_{2}\right)}=\frac{R_{4}}{R_{3}+R_{4}} \\
& \Rightarrow \frac{j \omega R_{2} C_{1}}{\left(1+j \omega R_{1} C_{1}\right)\left(1+j \omega R_{2} C_{2}\right)+j \omega R_{2} C_{1}}=\frac{R_{4}}{R_{3}+R_{4}} \\
& \Rightarrow \frac{j \omega R_{2} C_{1}}{\left(1+j \omega R_{1} C_{1}\right)\left(1+j \omega R_{2} C_{2}\right)+j \omega R_{2} C_{1}}=\frac{R_{4}}{R_{3}+R_{4}} \\
& \Rightarrow \frac{j \omega R_{2} C_{1}}{\left(1+j \omega R_{1} C_{1}+j \omega R_{2} C_{2}-\omega^{2} R_{1} C_{1} R_{2} C_{2}\right)+j \omega R_{2} C_{1}}=\frac{R_{4}}{R_{3}+R_{4}} \\
& \Rightarrow \frac{j \omega R_{2} C_{1}}{\left(1-\omega^{2} R_{1} C_{1} R_{2} C_{2}\right)+j \omega\left(R_{1} C_{1}+R_{2} C_{2}+R_{2} C_{1}\right)}=\frac{R_{4}}{R_{3}+R_{4}} \tag{1}
\end{align*}
$$

Since, the right hand side of the above equation is a real term, the left hand side must also be a real term. So, we can write,
$1-\omega^{2} \mathrm{R}_{1} \mathrm{C}_{1} \mathrm{R}_{2} \mathrm{C}_{2}=0$
$\Rightarrow \omega=\frac{1}{\sqrt{\mathrm{R}_{1} \mathrm{C}_{1} \mathrm{R}_{2} \mathrm{C}_{2}}} \ldots \ldots$
From equation (1) we have,

$$
\begin{aligned}
& \Rightarrow \frac{\mathrm{R}_{2} \mathrm{C}_{1}}{\left(\mathrm{R}_{1} \mathrm{C}_{1}+\mathrm{R}_{2} \mathrm{C}_{2}+\mathrm{R}_{2} \mathrm{C}_{1}\right)}=\frac{\mathrm{R}_{4}}{\mathrm{R}_{3}+\mathrm{R}_{4}} \\
& \Rightarrow \frac{\mathrm{R}_{2} \mathrm{C}_{1}}{\mathrm{R}_{1} \mathrm{C}_{1}+\mathrm{R}_{2} \mathrm{C}_{2}}=\frac{\mathrm{R}_{4}}{\mathrm{R}_{3}}
\end{aligned}
$$

$\Rightarrow \frac{\mathrm{R}_{3}}{\mathrm{R}_{4}}=\frac{\mathrm{R}_{1} \mathrm{C}_{1}+\mathrm{R}_{2} \mathrm{C}_{2}}{\mathrm{R}_{2} \mathrm{C}_{1}}=\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}+\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}$
The op-amp along with the two resistors $\mathrm{R}_{3}$ and $\mathrm{R}_{4}$ constitutes a non-inverting amplifier who's voltage gain is,
$A_{v}=1+\frac{R_{3}}{R_{4}}$
Using the of $\mathrm{R}_{3} / \mathrm{R}_{4}$ obtained in equation (3) we have,
$A_{v}=1+\frac{R_{1}}{R_{2}}+\frac{C_{2}}{C_{1}}$
This corresponds that the attenuation of the feedback network is,
$1 /\left(1+\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}+\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}\right)$
Therefore, $A_{V}$ must be equal to or greater than $\left(1+\frac{R_{1}}{R_{2}}+\frac{C_{2}}{C_{1}}\right)$ to sustain oscillation.
Mathematically,
$\mathrm{A}_{\mathrm{v}} \geq\left(1+\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}+\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}\right) \ldots \ldots$
$\Rightarrow 1+\frac{\mathrm{R}_{3}}{\mathrm{R} 4} \geq\left(1+\frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}+\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}\right)$
$\Rightarrow \frac{\mathrm{R}_{3}}{\mathrm{R}_{4}} \geq \frac{\mathrm{R}_{1}}{\mathrm{R}_{2}}+\frac{\mathrm{C}_{2}}{\mathrm{C}_{1}}$
For $\mathrm{R}_{1}=\mathrm{R}_{2}=\mathrm{R}$ and $\mathrm{C}_{1}=\mathrm{C}_{2}=\mathrm{C}$ we have,
$\frac{\mathrm{R}_{3}}{\mathrm{R}_{4}} \geq 2$
$\Rightarrow \mathrm{R}_{3} \geq 2 \mathrm{R}_{4}$
Also from equation (4) we have, $\mathrm{A}_{\mathrm{v}} \geq 3$

From equation (2) we have,
$\omega=\frac{1}{\mathrm{RC}}$
$\Rightarrow \mathrm{f}=\frac{1}{2 \pi \mathrm{RC}}$


## Example: Design of Wein Bridge Oscillator

Design the Wein bridge oscillator to produce a 100 kHz output frequency with an amplitude of $\pm 9 \mathrm{~V}$. Design the amplifier to have a closed-loop gain of 3 .

## SOLUTION $\quad V_{\mathrm{CC}}= \pm\left(V_{0}+1 \mathrm{~V}\right)= \pm(9 \mathrm{~V}+1 \mathrm{~V})= \pm 10 \mathrm{~V}$

For $A_{\mathrm{CL}}=3$,

$$
R_{1}=R_{2}=R \text { and } C_{1}=C_{2}=C
$$

Also,

$$
R_{3}=2 R_{4}
$$

Select,

$$
C_{1}=1000 \mathrm{pF} \text { (standard value) }
$$

$$
C_{2}=C_{1}=1000 \mathrm{pF}
$$

Therefore, $R=\frac{1}{2 \pi f C}=\frac{1}{2 \pi \times 100 \mathrm{kHz} \times 1000 \mathrm{pF}}$

$$
=1.59 \mathrm{k} \Omega \text { (use } 1.5 \mathrm{k} \Omega \text { standard value) }
$$

Select, $\quad R_{4} \approx R_{2}=1.5 \mathrm{k} \Omega$ (standard value)

$$
R_{3}=R_{4}=2 \times 1.5 \mathrm{k} \Omega=3 \mathrm{k} \Omega \text { (use } 3.3 \mathrm{k} \Omega \text { standard value) }
$$

The OP-AMP full-power bandwidth $\left(f_{\mathrm{p}}\right)$ must be a minimum of 100 kHz when $V_{0} \approx \pm 9 \mathrm{~V}$ and $A_{\mathrm{CL}}=3$.

Since $f_{2}=A_{\mathrm{CL}} \times f_{\mathrm{p}}$, therefore,

$$
f_{2}=3 \times 100 \mathrm{kHz}=300 \mathrm{kHz}
$$

and

$$
\text { Slew rate, } S R=2 \pi f_{\mathrm{p}} V_{\mathrm{p}}=2 \pi \times 100 \mathrm{kHz} \times 9 \mathrm{~V} \approx 5.7 \mathrm{~V} / \mu s
$$



Figure 9.25 A piezoelectric crystal: (a) Symbol, (b) electrical model, and (c) the reactance function (if $R=0$ ).

### 9.12 CRYSTAL OSCILLATORS

If a piezoelectric crystal, usually quartz, has electrodes plated on opposite faces and if a potential is applied between these electrodes, forces will be exerted on the bound charges within the crystal. If this device is properly mounted, deformations take place within the crystal, and an electromechanical system is formed which will vibrate when properly excited. The resonant frequency and the $Q$ depend upon the crystal dimensions, how the surfaces are oriented with respect to its axes, and how the device is mounted. Frequencies ranging from a few kilohertz to a few megahertz, and $Q$ 's in the range from several thousand to several hundred thousand, are commercially available. These extraordinarily high values of $Q$ and the fact that the characteristics of quartz are extremely stable with respect to time and temperature account for the exceptional frequency stability of oscillators incorporating crystals. Crystal oscillators are used whenever great stability is required, for example, in communication transmitters and receivers.

The electrical equivalent circuit of a crystal is indicated in Figure 9.25. The inductor $L$, capacitor $C$, and resistor $R$ are the analogs of the mass, the compliance (the reciprocal of the spring constant), and the viscous-damping factor of the mechanical system. The typical values for a $90-\mathrm{kHz}$ crystal are $L=137 \mathrm{H}, C=0.0235 \mathrm{pF}$, and $R=15 \mathrm{k} \Omega$, corresponding to $Q=5,500$. The dimensions of such a crystal are 30 by 4 by 1.5 mm . Since $C^{\prime}$ represents the electrostatic capacitance between electrodes with the crystal as a dielectric, its magnitude ( $\sim 3.5 \mathrm{pF}$ ) is very much larger than $C$.

If we neglect the resistance $R$, the impedance of the crystal ( $Z$ in Figure $9.25(\mathrm{~b})$ ) is given by,

$$
\begin{array}{rlrl} 
& & Z= & Z_{1} \| Z_{2} \\
\Rightarrow & & =\frac{Z_{1} Z_{2}}{Z_{1}+Z_{2}} \\
\Rightarrow & & =\frac{\left(-j 1 / \omega C^{\prime}\right)(j \omega L-j 1 / \omega C)}{j \omega L-j 1 / \omega C-j 1 / \omega C^{\prime}} \\
\Rightarrow & & =-\frac{j}{\omega C^{\prime}} \frac{L(\omega-1 / \omega L C)}{L\left(\omega-1 / \omega L C-1 / \omega L C^{\prime}\right)} \\
\Rightarrow & & =-\frac{j}{\omega C^{\prime}} \frac{\omega^{2}-1 / L C}{\omega^{2}-\frac{1}{L}\left(1 / C+1 / C^{\prime}\right)}
\end{array}
$$

$$
\begin{equation*}
\Rightarrow \quad=-\frac{j}{\omega C^{\prime}} \frac{\omega^{2}-\omega_{\mathrm{s}}^{2}}{\omega^{2}-\omega_{\mathrm{p}}^{2}} \tag{9.64}
\end{equation*}
$$

Where $\omega_{\mathrm{s}}=\frac{1}{\sqrt{L C}}=$ series resonant frequency and $\omega_{\mathrm{p}}=\sqrt{\frac{1}{L}\left(\frac{1}{C}+\frac{1}{C^{\prime}}\right)}=$ parallel resonant frequency. Equation (9.64) can be written, in terms of reactance, as

$$
\begin{equation*}
Z=j X=-\frac{j}{\omega C^{\prime}} \frac{\omega^{2}-\omega_{\mathrm{s}}^{2}}{\omega^{2}-\omega_{\mathrm{p}}^{2}} \tag{9.65}
\end{equation*}
$$

Therefore, reactance of the crystal is

$$
\begin{equation*}
X=-\frac{1}{\omega C^{\prime}} \frac{\omega^{2}-\omega_{\mathrm{s}}^{2}}{\omega^{2}-\omega_{\mathrm{p}}^{2}} \tag{9.66}
\end{equation*}
$$

The plot of Equation (9.66) is shown in Figure $9.25(\mathrm{c})$. Since $C^{\prime} \gg C, \omega_{\mathrm{p}} \approx \omega_{\mathrm{s}}$. For the crystal whose parameters are specified above, the parallel frequency is only three-tenths of 1 percent higher than the series frequency. For $\omega_{\mathrm{s}}<\omega<\omega_{\mathrm{p}}$, the reactance is inductive, and outside this range it is capacitive, as indicated in Figure 9.25(c). In order to use the crystal properly it must be connected in a circuit so that its low impedance in the series resonant operating mode or high impedance in the parallel resonant operating mode is selected.

Figure 9.26
Crystal-controlled oscillator using crystal in series feedback path:
(a) BJT circuit, and (b) FET circuit.

(a)

(b)

## Series Resonant Circuits

To excite a crystal for operation in the series resonant mode it may be connected as a series element in a feedback path. At the series resonant frequency of the crystal its impedance is smallest and the amount of (positive) feedback is largest. A typical transistor circuit is shown in Figure 9.26. Resistors $R_{1}, R_{2}$, and $R_{\mathrm{E}}$ provide a voltage divider stabilized $d c$ bias circuit.

Capacitor $C_{\mathrm{E}}$ provides $a c$ bypass of the emitter resistor and the $R F C$ coil provides for $d c$ bias while decoupling any ac signal on the power lines from affecting the output signal. The voltage feedback from collector to base is a maximum when the crystal (XTAL) impedance is minimum (in series resonant mode). The coupling capacitor $C_{\mathrm{C}}$ has negligible impedance at the circuit operating frequency but blocks ant dc between collector and base.

The resulting circuit frequency of oscillation is set by the series resonant frequency of the crystal. Changes in supply voltage, transistor device parameters, and so on, have no effect on the circuit operating frequency which is held stabilized by the crystal. The circuit frequency stability is set by the crystal frequency stability, which is good.

Figure 9.27
Crystal-controlled oscillators in parallel resonant operating mode: (a) BJT circuit, and (b) FET circuit.

(a)

(b)

## Parallel Resonant Circuits

Since the parallel resonant impedance of a crystal is a maximum value, it is connected in parallel/shunt. At the parallel resonant operating frequency a crystal appears as an inductive reactance of largest value. Figure 9.27(a) shows a crystal connected as the inductor connected in a modified Colpitts circuit. The basic $d c$ bias circuit should be evident. Maximum voltage is developed across the crystal at its parallel resonant frequency. The voltage is coupled to the emitter by a capacitor voltage divider - capacitors $C_{1}$ and $C_{2}$.

A Miller crystal-controlled oscillator circuit is shown in Figure 9.27(b). A tuned LC circuit in the drain section is adjusted near the crystal parallel resonant frequency. The maximum gate-source signal occurs at the crystal parallel resonant frequency controlling the circuit operating frequency.

## OP-AMP Crystal Oscillator

An OP-AMP can be used in a crystal oscillator as shown in Figure 9.28. The crystal is connected in the series resonant path and operates at the crystal series resonant frequency. The present circuit has a high gain so that an output square-wave signal results as shown in the figure. A pair of Zener diodes is shown at the output to provide output amplitude at exactly the Zener voltage $\left(V_{\mathrm{Z}}\right)$.


Figure 9.28 Crystal oscillator using OP-AMP.

